# FOUR COMPARATIVE STEADY-STATE EXERCISES USING THE DIAMOND-MORTENSEN-PISSARIDES MODEL: EMPIRICAL- AND POLICY-DRIVEN APPLICATIONS FOR FIRST-YEAR GRADUATE STUDENTS IN MACROECONOMICS

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**Abstract**

The Diamond-Mortensen-Pissarides (DMP) model of two-sided labor market search is now a common topic in first-year graduate courses in macroeconomics. Following Pissarides (2011), Alogoskoufis (2019) derives a two-equation in two unknowns solution to the DMP model that is useful for teaching this topic. I discuss four comparative steady-state exercises that he does not consider: (i) a direct change in the posting price of a vacancy; (ii) a change in labor-market matching efficiency; (iii) financing unemployment benefits that are proportional to the real wage with a labor income tax; and (iv) the addition of a minimum wage. The first two exercises are empirically motivated, and the second two exercises are policy motivated.

Keywords: two-sided labor market search; comparative steady-state exercises; graduate teaching

JEL Codes: A23, E24

## Introduction

Dynamic labor-market search models with frictions such as the Diamond-Mortensen-Pissarides (DMP) model are now a common topic in first-year graduate macroeconomics courses.[2,3] While there are several treatments of this model that one can use as a basis of instruction, I find the approach that Alogoskoufis (2019, ch. 18) takes, which is based upon Pissarides (2011), to be particularly useful. I use Alogoskoufis's solution as a starting point for analyzing four comparative steady state exercises that his discussion of the DMP model does not cover. I begin with a brief description of the Pissarides-Alogoskoufis solution of the DMP model. I then present my four comparative steady-state equilibrium exercises and briefly discuss why they are of empirical and/or policy interest. The paper concludes with a brief summary.

## The Pissarides-Alogoskoufis Solution to the DMP Model

Following Pissarides (2011), Alogoskoufis (2019) starts with Bellman equations for an employed and an unemployed consumer, for a firm with a filled position and with a vacancy, as

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[2] As an example, I have taught this subject in a class comprised of both first-year MA and PhD students, with differing levels of preparation in calculus, for the past 13 years.

[3] Key early contributions include Diamond (1982), Pissarides (1985), Mortensen (1986), and Mortensen and Pissarides (1994). For a historical overview of the DMP model, see Pissarides (2011).

well as the Nash bargaining game and its first-order condition that the real wage must solve. From these five equations, Alogoskoufis derives two equations for the real wage, $w$, as a function of labor-market tightness, $\theta = \frac{v}{u}$, where $u$ is the unemployment rate and $v$ is the vacancy rate. Specifically, letting $m(u,v)$ be the usual constant returns to scale matching function, he shows that these two relationships satisfy:

$$w = p - (r+\delta)\frac{k}{m(1/\theta,1)} \tag{1}$$

$$w = \beta p + (1-\beta)z + \beta k\theta \tag{2}$$

where I have maintained Alogoskoufis's notation save for denoting the posting price of a vacancy with $k$, rather than his $pc$, and the probability of a filled vacancy, $m(u,v)/v$, with $m(1/\theta,1)$, rather than his $q(\theta)$.[4] I opt to make these two changes in notation for the following reasons. First, in my view it is not *a priori* obvious that the posting price is necessarily proportional to labor productivity so assuming instead that it equals a constant takes a more neutral stand on any such relationship. Second, I also believe that by writing $m(1/\theta,1)$ and $m(1,\theta)$ for the probabilities of a firm filling a vacancy and a searching consumer being hired makes the relationship between each of these functions and labor-market tightness more apparent than does writing them as $q(\theta)$ and $\theta q(\theta)$.

Pissarides (2011) and Alogoskoufis (2019) refer to equation (1) as the *job creation condition* and to equation (2) as the *wage determination equation*. The former is a decreasing function and the latter is an increasing function of labor-market tightness. Note that the equilibrium is unique and strictly positive for both $\theta$ and $w$ if $p-(r+\delta) > \beta p + (1-\beta)z$ which is assumed.

To close the model, with the equilibrium values of the real wage and labor-market tightness now determined, it follows that the steady-state unemployment and vacancy rates are constant over time. Focusing on the unemployment rate, it follows that the flows of consumers into and out of employment must be equal:

$$u^* m(1,\theta^*) = (1-u^*)\lambda \tag{3}$$

Solving this for $u^*$ yields $u^* = \frac{\lambda}{\lambda + m(1,\theta^*)}$ and from the definition of $\theta$ we obtain $v^* = \frac{\theta^*\lambda}{\lambda + m(1,\theta^*)}$.

In my view, one of the primary advantages of presenting the DMP model using the Pissarides-Alogoskoufis approach is that the model directly solves for the equilibrium real wage, a variable that is of particular interest when considering the effects of changes in the parameters on the model's steady-state equilibrium. Alogoskoufis (2019) considers several such exercises including (i) an increase in marginal product of labor, $p$, which necessarily implies an increase in the posting price of a vacancy as the latter equals $pc$; (ii) an increase in the unemployment benefit, $z$; (iii) an increase in the relative bargaining power of consumers, $\beta$; (iv) an increase in the real interest rate, $r$; and (v) an increase in the separation rate, $\lambda$. With regards to (ii), he also considers the case where the unemployment benefit is the share $\rho$ (the replacement rate) of the real wage so that $z = \rho w$.

[4] Specifically, $p$ is the constant marginal product of labor; $z$ is the unemployment benefit; $\lambda$ is the separation rate; $\beta$ is the relative bargaining power of the consumer in the Nash bargaining game; and $r$ is the real rate of interest.

**Four Steady-State Equilibrium Exercises**

When I teach the DMP model, besides discussing the aforementioned exercises, I also include four others that Alogoskoufis (2019) does not discuss. The first exercise that I present, and one that is a relatively simple starting place, is a change in the posting price, *k*. Innovations in the technology for the posting of vacancies such as the online job sites that employers provide or more general online job sites such as *Indeed* have substantially reduced the cost to firms of doing so. As expected, when the posting price decreases, the job creation condition increases causing both labor-market tightness and the real wage to increase. The lower posting price increases the number of posted positions and this in turn increases the probability that a searching consumer is matched. Thus, the unemployment rate decreases and the vacancy rate increases.

Second, following Williamson (2018), I modify the matching function to include a term that captures the efficiency of the matching process. Thus, I write the matching function as $em(u,v)$ where *e* measures the efficiency of the labor-market matching process. Since only equation (1) includes the parameter *e* through what is now the $em(1/\theta,1)$ term, it is straightforward to determine the impact of a change in labor-market matching efficiency on the equilibrium values of $w^*$ and $\theta^*$ and hence on $u^*$ and $v^*$. Assuming, as seemingly occurred during the Great Recession, that *e* decreased, then it follows that equation (1) decreases so that both $w^*$ and $\theta^*$ decline. The decrease in labor-market tightness and the decrease in matching efficiency both serve to increase the unemployment rate since we now have that $u^* = \frac{\lambda}{\lambda + em(1,\theta^*)}$. As for the vacancy rate, the decrease in matching efficiency can cause it to *increase* rather to decrease. Specifically, this is the case if the absolute value of the elasticity of the unemployment rate with respect to labor-market matching efficiency is sufficiently small.[5] This result provides a possible (and plausible) explanation for the rightward shift in the US Beveridge curve observed during and after the Great Recession. Moreover, it is also consistent with the second rightward shift in the curve observed during and after the COVID-19 recession.

Third, while Alogoskoufis (2019) considers the effects from a change in unemployment benefits, be they exogenous or endogenous, he does not consider the general equilibrium question of how to finance these benefits. My approach is to assume that employed consumers face a proportional tax on their wage income and that this revenue finances unemployment benefits that are proportional to the real wage. Thus, the wage determination equation now reads:

$$(1-\tau)w = \beta p + (1-\beta)\rho w + \beta k\theta \tag{4}$$

where $\tau$ is the labor- income tax rate and $\rho$. is the replacement rate. Since the measure of employed consumers equals $m(u,v)$, each of whom is taxed $\tau w$, and the measure of unemployed consumers equals *u*, each of whom receives an unemployment benefit of $z = \rho w$, it follows that the government budget constraint in each period satisfies

$$m(u,v)\tau w = u\rho w \tag{5}$$

[5] Recall that $v^* = \theta^* u^*$ so that $\frac{dv^*}{de} = \theta \frac{du^*}{de} + u^* \frac{d\theta}{de} > 0$ if and only if $\varepsilon_{\theta,e} > -\varepsilon_{u,e} > 0$ where $\varepsilon_{x,e}$ is the elasticity of *x* with respect to *e*.

Dividing both sides by $w$ and then by $u$ implies that $m(1,\theta)\tau = \rho$. Substituting this expression into equation (4) and rearranging yields the *wage determination equation* for this version of the model:

$$w = \frac{\beta(p + k\theta)}{1 - \tau[1 + (1 - \beta)m(1,\theta)]} \tag{6}$$

Notice that equation (6) continues to be an increasing function of labor-market tightness. Combining this expression with the job creation condition allows one to solve the model in the usual way. In doing so, we can now study, for example, the impact of a change in the tax rate on the steady-state equilibrium. An increase in the labor-income tax rate increases equation (6) which in turn causes the before-tax real wage to increase and labor-market tightness to decrease as firms are now less willing to post vacancies. Hence, the unemployment rate increases, and the vacancy rate decreases. As for the replacement rate, $\rho = \tau m(1,\theta)$, it can either increase or decrease depending upon the usual Laffer curve tradeoff between the tax rate, $\tau$, and the effective tax base, $m(1,\theta)$. For values of $\tau$ near zero, we would expect that an increase in the tax rate would reduce the probability of employment by a smaller proportion thereby causing the replacement rate to increase.[6]

The fourth exercise that I cover is the introduction of a minimum wage. The International Labour Organization (ILO) reports that 92% of its 186 member states (as of 2015) have a minimum wage in place and so such laws are quite common in practice (ILO 2017, Ch 1.2). Consequently, analyzing the DMP model in the presence of a minimum wage is of interest in and of itself as are analyzing the effects of a change in the minimum wage or how changes in other parameters affect the steady state when a minimum wage exists.

The presence of a minimum wages adds a constraint to the Nash bargaining game such that now we have:

$$w^* = \arg\max \left(W(w) - U\right)^{\beta} \left(J(p - w) - V\right)^{1-\beta} \text{ subject to } w \geq \underline{w} \tag{7}$$

where $\underline{w}$ is the minimum wage and $W$, $U$, $J$, and $V$ are the value functions for an employed consumer, an unemployed consumer, a firm with a filled position, and a firm with a vacancy. Since Nash bargaining does not enter into the derivation of the job creation condition, equation (1) is unchanged. However, because the Nash bargaining solution does enter into the derivation of the wage determination equation, we have to adjust it to allow for the case where the minimum wage is binding. Therefore, we replace equation (2) with:

$$\begin{aligned} w &= \beta p + (1-\beta)z + \beta k\theta \quad w^* > \underline{w} \\ w &> \beta p + (1-\beta)z + \beta k\theta \quad w^* = \underline{w} \end{aligned} \tag{8}$$

If the minimum wage is non-binding in equilibrium, then equation (8) corresponds to equation (2) and so is increasing in $\theta$ as usual. If the minimum wage is binding in equilibrium, then the wage

[6] Alternatively, one can use equation (5) to solve for $\tau$ in term of $\rho$ and then consider the general equilibrium effects of a change in the replacement rate on the labor-income tax rate.

determination equation is constant at $w^* = \underline{w}$ and exceeds the section of the wage determination equation that would otherwise apply for $\theta < \theta^*$. Figures 1 and 2 illustrate the two cases, non-binding and binding, respectively.

Figure 1: The case of an equilibrium with a non-binding minimum real wage.

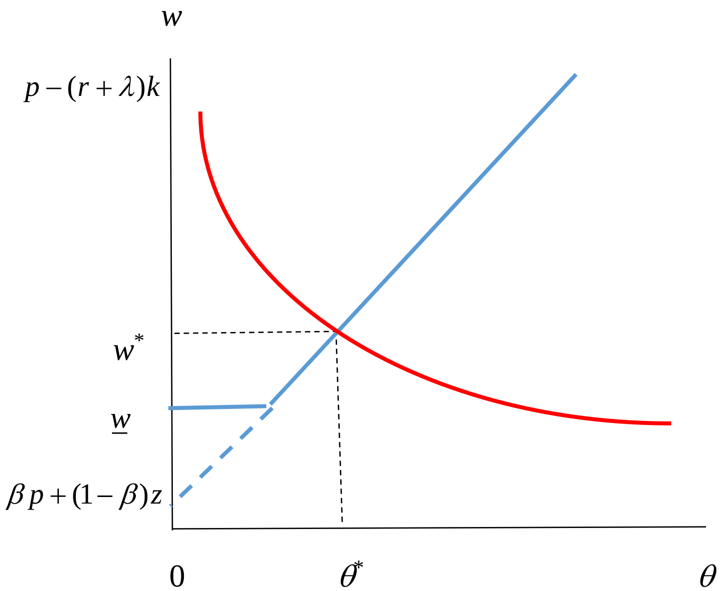
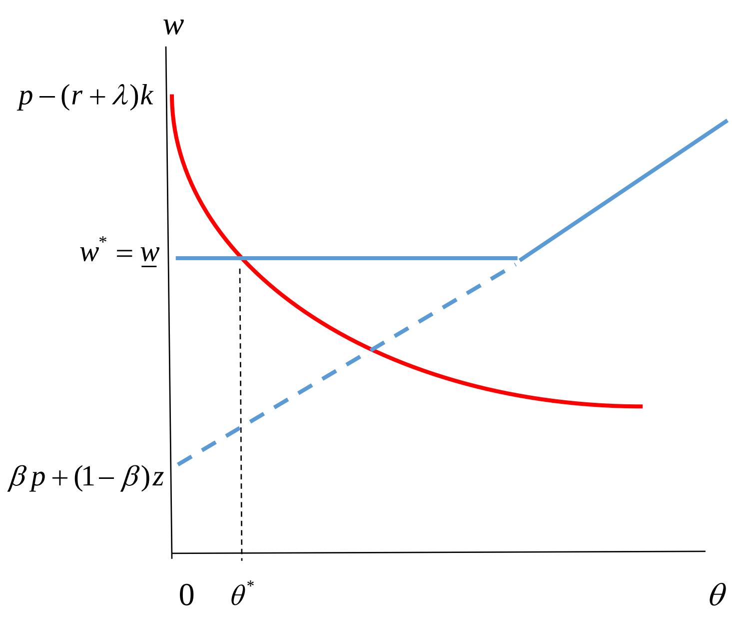


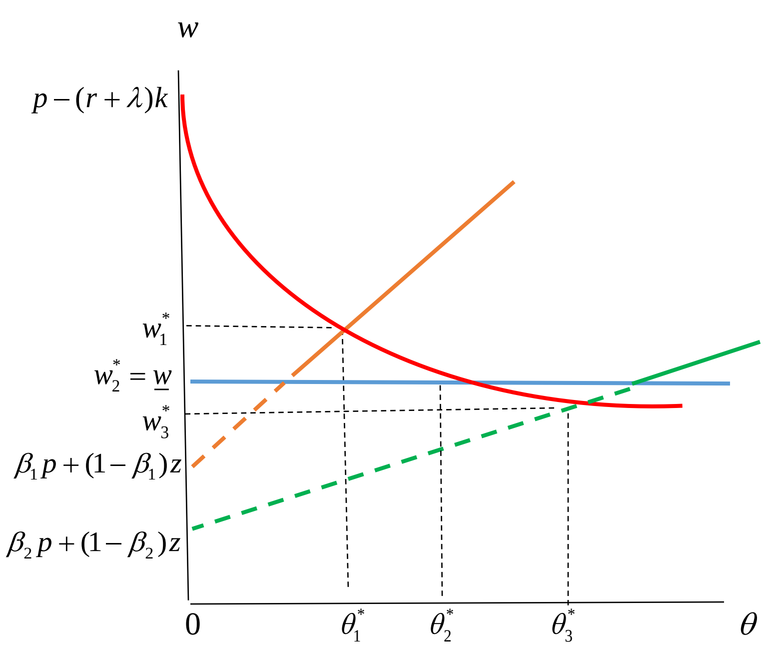

Figure 2: The case of an equilibrium with a binding minimum real wage.




One can use this extension to investigate the impact of an increase in the minimum wage when the minimum wage is currently non-binding versus is currently binding. At the margin, while the former is irrelevant, the latter necessarily implies that labor-market tightness decreases. The higher binding real wage reduces the incentive for firms to post vacancies and raises the incentive for consumers to search for positions. Therefore, the vacancy rate decreases, and the unemployment rate increases.

Another line of inquiry is to illustrate how changes in the other parameters of the model have the potential to move the steady-state equilibrium from one that is non-binding to one that is binding. For example, a decrease in the relative bargaining power of consumers, $\beta$, causes the wage determination equation to decrease (both its vertical intercept and its slope decrease) which in turn increases the level of labor-market tightness at which the minimum wage binds. When the decrease is sufficiently large, in the new steady state, firms prefer to reduce the real wage below $\underline{w}$, but cannot do so. However, with the real wage now lower than it was in the initial steady state, firms are more willing to post vacancies causing labor-market tightness to increase, albeit not as much as it would have increased were there no minimum wage. With more firms posting vacancies, the vacancy rate increases, but by less than without the minimum wage constraint. Similarly, the unemployment rate decreases, but by less than without the constraint. Figure 3 illustrates this example wherein the consumer's relative bargaining power decreases from $\beta_1$ to $\beta_2$ and labor-market tightness increases from $\theta_1^*$ to $\theta_2^*$, a smaller increase than would have occurred in the absence of the minimum wage, a level denoted by $\theta_3^*$.

Figure 3: A decrease in the relative bargaining power of consumers from $\beta_1$ to $\beta_2$ causes the equilibrium real wage to decline from $w_1^*$ to $w_2^* = \underline{w}$, the minimum wage, and labor-market tightness to increase from $\theta_1^*$ to $\theta_2^*$. In the absence of the minimum wage, labor-market tightness would increase further to $\theta_3^*$ and the equilibrium real wage would fall to $w_3^* < \underline{w}$.




**Conclusion**

Given that teaching the DMP model in first-year graduate courses in macroeconomics is now quite common, from an instructional viewpoint it is desirable to have a relatively simple version of the model that one can present to students, especially master's-level students who may not have more than a semester of calculus in their background. Pissarides (2011) and Alogoskoufis (2019) present such a version of the DMP model. While Alogoskoufis provides several interesting comparative steady state examples, in my first-year graduate macroeconomics class I also discuss the four exercises considered above. In my view, these exercises provide theoretical insights into certain empirical results (falling costs to post vacancies; the shift in the US Beveridge curve) and certain questions of macroeconomic policy (changes in the funding of unemployment benefits; changes in the minimum wage). I believe that these four exercises provide interesting theoretical applications of the DMP model, each of which is grounded in either empirics or policy. I offer them in the hope that they will be of interest to instructors of a first-year graduate macroeconomics course who cover the DMP model.

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