

## The AD-AS Model with the Shadow Economy

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### Abstract

This theoretical paper includes the shadow economy in the popular AD-AS model and derives the main economic results for both the short-run and the long-run. This accounts for the presence of the shadow economy in countries around the world, as well its persistence over time. Precisely, if the shadow economy is capable of absorbing unemployment, then in the short-run there will be a supply-side positive shock on output (the “shadow shock”). The “shadow shock” is, however, temporary; in the long-run the effect of the shadow economy on potential output is negative.

Key words: economic education, teaching of economics, general aggregative models, shadow economy.

JEL Classification: A2, E10, E26, O17

### Introduction

The Aggregate Demand-Aggregate Supply (hereinafter, AD-AS) model is one of the fundamental tools in economics. It is probably the most popular and the most studied at the undergraduate level, because it allows economists to analyse – in a straightforward graphical representation – the effects of economic shocks and economic policies on three key macroeconomic variables: growth, inflation, and unemployment. Also, the AD-AS model is a very general and flexible framework, since it accommodates both the Keynesian view (that focuses on aggregate demand over the short-run) and the Classical approach (that focuses on aggregate supply over the long-run).

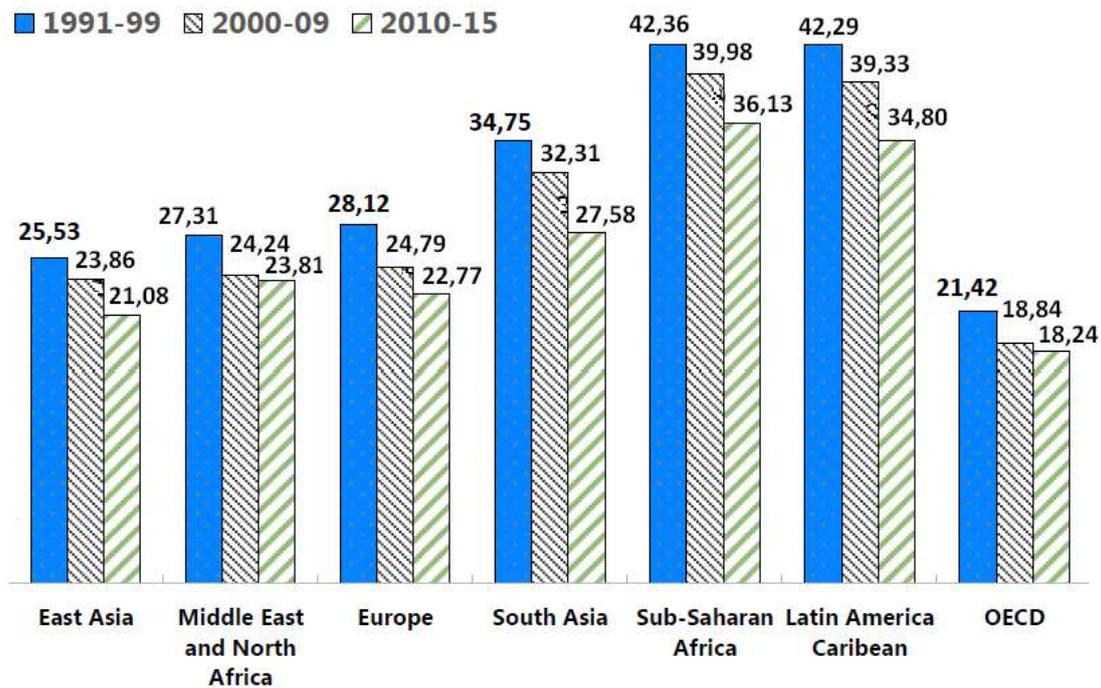
However, while the factors which determine economic growth (such as technological progress and human capital) indirectly appear in the AD-AS model (in determining the potential output), another important economic phenomenon, the shadow economy, finds no place in this very widespread and well-studied economic model.

The **shadow economy** (also known as hidden economy, black economy, underground economy, and informal economy) refers to all economic activities that are hidden to official authorities for various reasons (such as avoiding paying taxes and social security contributions, or escaping governmental bureaucracy and regulatory burdens). The shadow economy is, by definition, a phenomenon that is difficult to measure, since it includes different types of activities: legal productive activities, illegal and criminal activities, do-it-yourself and household activities. Furthermore, in macroeconomic estimates, it is often difficult to distinguish between legal

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and illegal activities.<sup>2</sup> This is a very important shortcoming, since only legal activities contribute (if recorded) to the national wealth, thereby, increasing taxable income.



**Figure 1.** *Shadow Economy over time and by Region* (average, percent of Gross Domestic Product. Source: Medina and Schneider, 2017)

Figure 1 reports recent estimates of the shadow economy for 158 Countries over time for 1991-2015. Two clear results emerge from Figure 1. First, the shadow economy is a widespread phenomenon around the world, albeit to varying extents. Second, the shadow economy is a phenomenon that persists over time. This topic, therefore, deserves to be included in the most popular and most studied macroeconomic model. Indeed, the main aim of this theoretical paper is to incorporate the shadow economy into the AD-AS model and to derive the main economic results, in both the short-run and the long-run, thus making this popular but “old” framework more alive.

### Aggregate Demand with tax evasion

The AD-AS model aims at explaining one of the most important topics in macroeconomics: short-run fluctuations over the so-called business cycle. These are the deviations of the actual

<sup>2</sup> The size of a shadow economy has many causes, including tax burden, corruption, organised crime, government instability, low quality of political institutions, and weak rule of law (see, e.g. Medina and Schneider, 2017).

(real) Gross Domestic Product (GDP) from its potential or long-run level.<sup>3</sup> Mathematically, the AD-AS model consists of two functions: Aggregate Demand and Aggregate Supply.

The Aggregate Demand function (hereinafter, AD) represents the aggregate expenditure of an economic system, namely, the sum of households' consumption ( $C$ ), firms' investment ( $I$ ) and public spending ( $G$ ):<sup>4</sup>

$$AD = C + I + G \quad (1)$$

The AD is a relationship between changes in a price index  $P$  (such as the Consumer Price Index or the GDP deflator) and the amount of real GDP when the goods market and the financial markets are in equilibrium, namely, when aggregate expenditure is equal to real output (GDP), denoted by  $Y$ . At a lower price level, the same nominal amount of total spending will purchase more real GDP and vice versa. Therefore, the slope of the AD curve must be negative, in the tautological sense.

In order to highlight the key role of the shadow economy in equation (1), as well as the strong link between the shadow economy and **tax evasion** ( $E$ ), i.e. the illegal non-payment or underpayment of tax, a simple government budget constraint is introduced into the AD-AS model (in the textbooks,  $G$  is treated as an exogenous variable):

$$G = T \quad (2)$$

$$T = t \cdot (1 - \varphi) \cdot Y = t \cdot Y - E \quad (3)$$

$$E = t \cdot \varphi \cdot Y \quad (4)$$

where  $T$  is state revenue,  $0 < t < 1$  is the flat tax rate on output  $Y$ , and  $0 < \varphi < 1$  is the share of the shadow economy in total output. Of course, as highlighted by equation (4), there is a positive relation between the shadow economy and tax evasion. As the shadow economy cannot be taxed, an increase in  $\varphi$  (and thus in  $E$ ) reduces state revenues and, consequently, the quality and quantity of the supply of public goods and services (Torgler, 2007). Thus, public expenditure ( $G$ ) falls.

The government budget constraint used in this model is very standard and is identical to that of Mazhar and Méon (2017), except for the presence of seigniorage. Mazhar and Méon (2017), in fact, assume that a government has two instruments with which to finance a given level of public spending: a flat tax on output and seigniorage. This leads to a positive relation between the shadow economy and inflation. However, their budget constraint relies on the unrealistic possibility that a government can control monetary policy. That is, there is no independent central bank. In our case, therefore, the government finances a given level of public spending, especially with respect to taxes.<sup>5</sup> When  $\varphi$  increases,  $t$  should increase to preserve the existing level of public spending. Hence, the shadow economy can also increase taxation, thus reducing

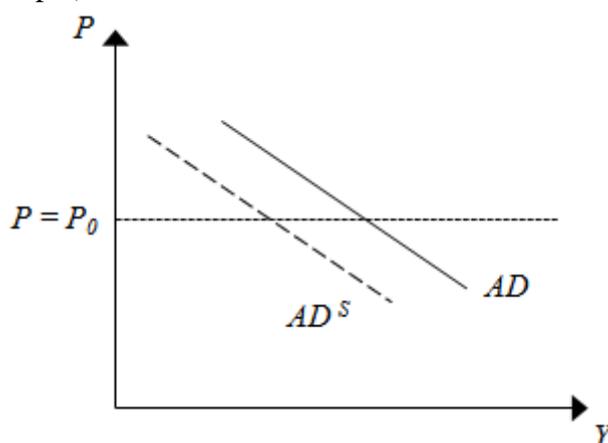
<sup>3</sup> For more details about the AD-AS model, see, e.g. Mankiw (2015).

<sup>4</sup> For the sake of simplicity, we consider an economy that is closed to foreign trade or, alternatively, an open economy where net exports (namely, exports minus imports) are always zero. Actually, the shadow economy is primarily a "domestic phenomenon", since in the case of foreign trade the possibility of being detected should increase.

<sup>5</sup> A realistic hypothesis, as in some countries such as Italy, where the shadow economy is a widespread phenomenon, the tax burden is heavy.

consumption. Nevertheless, aggregate consumption cannot increase, since the main beneficiaries of tax evasion are the tax evaders themselves (Alm and Finlay, 2013). That is, individuals with higher income and *ceteris paribus* lower marginal propensity to consumption.

Finally, investment can also decrease in the presence of the shadow economy. This is usually associated with corruption, organised crime, government instability, low quality of political institutions, and weak rule of law which can discourage investors or lead to a waste of resources (through bribes, for example).



**Figure 2.** The AD with the shadow economy and tax evasion

In short, all the main determinants of aggregate demand are negatively affected by the shadow economy. Hence, the AD with the shadow economy – which we call  $AD^S$  – is lower than the standard AD (see Figure 2), since *ceteris paribus* and for a given price index ( $P = P_0$ ), the aggregate expenditure of an economy with (a larger share of) the shadow economy is lower.

### Aggregate Supply with Shadow Employment

The Aggregate Supply function (hereinafter, AS) shows the quantity of goods and services supplied by an economy when there is equilibrium in the labour market. A simple equation is often used to describe the aggregate supply function:

$$AS = Y_p + \beta \cdot (P - P^E) \quad (5)$$

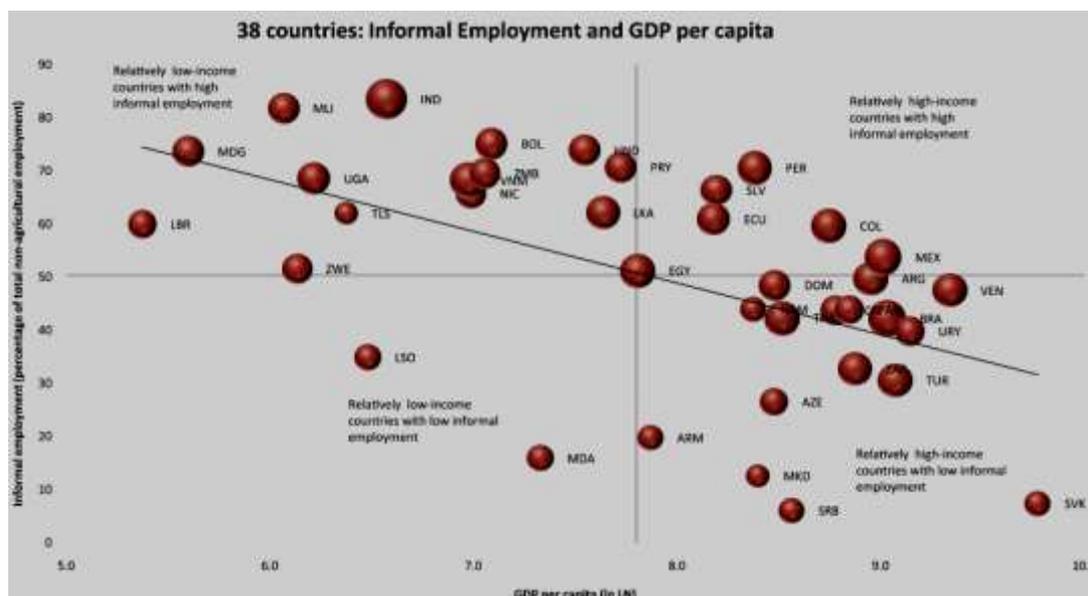
where  $\beta$  is a positive parameter ( $\beta > 0$ ),  $Y_p$  is the level of potential GDP, and  $P^E$  is price expectations. In the short-run, when price expectations are not correct ( $P \neq P^E$ ), the AS is upward sloping in the  $P$ - $Y$  space, meaning that when aggregate demand changes, firms adjust both price and quantity. For example, when aggregate demand increases, firms increase both price and quantity. In the long run, when price expectations are correct ( $P = P^E$ ), the Aggregate Supply function is a vertical straight line at  $Y = Y_p$ , the quantity of goods and services supplied by an economy coincides with the level of potential GDP.

In the long-run, economic theory agrees that the actual (real) GDP always coincides with the potential output of the economy ( $Y = Y_p$ ). Hence, economic policy has no real effects and an increase in aggregate demand only increases prices. In the short-run, however, there is a potential active role for economic policy: both government and the central bank can increase

the actual level of GDP (by means of expansive economic policies that move the aggregate demand curve to the right) at the cost of higher inflation (an increase in the percentage change in the price index  $P$ ).

In the AD-AS model, by definition, a higher level of potential GDP implies a lower natural unemployment rate. This is consistent with the empirical finding that (at least in advanced countries) productivity growth is strongly negatively correlated with unemployment in the long-run (see Pissarides and Vallanti, 2007).

In the shadow economy, firms employ mostly unskilled labour and adopt backward technology (see, e.g., Albrecht *et al.*, 2009; La Porta and Shleifer, 2008). Indeed, the shadow sector is very large in the poorest economies (see La Porta and Shleifer, 2008). Economic growth is, in fact, an essential element in reducing the shadow sector, although in some cases informality can persist, in spite of economic growth (see Figure 3).



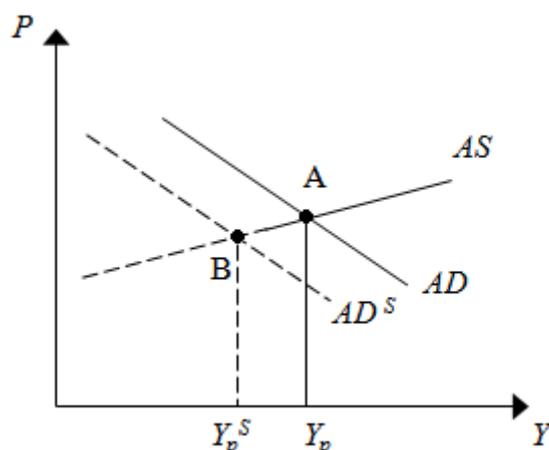
**Figure 3.** GDP per capita and informal employment (Source: ILO 2011)

Finally, the general finding of growth theory is that economic growth requires improvements in social infrastructure, a key component of which is government fiscal policy (Hall and Jones, 1999; Romer, 2006).<sup>6</sup> Thus, an economy with more tax evasion and a shadow economy is merely a society with poor social infrastructure.

Therefore, long-run growth and, thus, potential output should be higher in an economy without (or with a lower share of) a shadow economy, namely  $Y_p > Y_p^S$ , where  $Y_p^S$  denote the potential GDP of a society with (a larger share of) the shadow economy.

<sup>6</sup> According to the popular definition by Hall and Jones (1999), social infrastructures are “institutions and policies that align private and social returns.” In short, this would correctly or otherwise infer that in countries with good social infrastructure, negative phenomena (such as corruption and crime) are uncommon.

For the sake of simplicity, assume that the slope of the short-run AS curve (namely, the  $\beta$  parameter) does not change for a society with a shadow economy. Thus, the long-run equilibrium of the AD-AS model with a shadow economy can be portrayed as in Figure 4.



**Figure 4.** *The AD-AS model with the shadow economy in the long-run*

In Figure 4, point A (the intersection of AD and AS) characterises the long-run equilibrium of the standard version of the model; whereas, point B (the intersection of  $AD^S$  and AS) characterises the long-run equilibrium of the model with the shadow economy. By definition, in fact, the long-run equilibrium requires that actual GDP coincide with potential GDP.

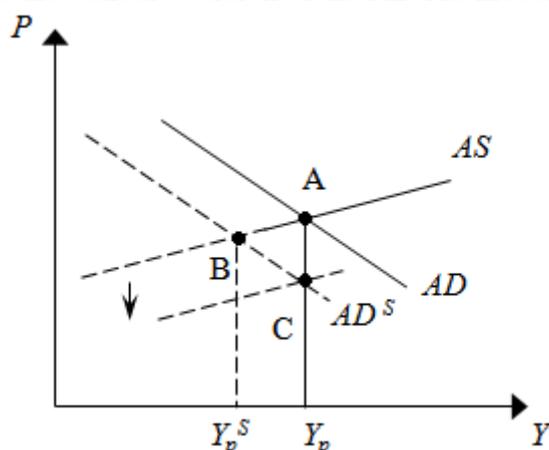
As previously stated, in the long-run the shadow economy has negative effects on potential GDP. In spite of that, the presence (in countries around the world) and the persistence (over time) of the shadow economy should involve the existence of (potential) positive effects on output, at least in the short-run.

It is noteworthy that unemployment may be absorbed by shadow employment (see, e.g. Boeri and Garibaldi, 2002, 2005; La Porta and Shleifer, 2008). **Shadow employment** (or undeclared work) refers to the use of labour without an official work contract. Note that, registered (official) firms can also make use of shadow employment. Firms that operate simultaneously both in the regular and in the shadow economy (by using undeclared work) are termed “moonlighting firms” (Busato et al., 2005; Ciccarone et al., 2012). In this context, if firms (for fiscal and economic reasons) increase the use of undeclared work and, thus, shadow employment is capable of absorbing unemployment, there could be a positive effect on both employment and output, thereby engendering a kind of supply-side positive shock, or “**shadow shock**”. If this happens, the short-run AS curve in the presence of the shadow economy moves downward in the  $P$ - $Y$  space and, thus,  $Y > Y_p^S$ . Considering equation (5), therefore, the “shadow shock” is associated with a decrease in  $P^E$ . By using shadow employment, indeed, firms expect to reduce their labor cost.

This completes the extension of the AD-AS model. Now, the main economic results are derived for both the short-run and the long-run.

### The AD-AS Model with or without the Shadow Economy: A Comparison

We compare two equilibria that only differ with respect to the share of the shadow economy, since the comparative statics of the model (the effect of changes in exogenous variables) do not change with respect to the standard version of the AD-AS model.



**Figure 5.** *The AD-AS model with and without the shadow economy: short-run and long-run*

For the sake of comparison, there are no other economic shocks aside from the shadow economy. Thus, in an economy without a shadow economy ( $\varphi = 0$ ), the long-run equilibrium coincides with the short-run equilibrium (point A in Figure 5).

As mentioned above, in an economy with tax evasion and shadow employment ( $0 < \varphi < 1$ ) there could be a positive economic shock on the supply side (the “shadow shock”). This “shadow shock” is, by definition, temporary, since there is no change in the potential level of GDP. Thus, point C in Figure 5 identifies the short-run equilibrium of the AD-AS model with the shadow economy (the short-run AS curve in the presence of the shadow economy – the dotted line in Figure 5 – moves downward).

In the short-run, therefore, in the presence of tax evasion and shadow employment (point C in Figure 5), the effect on output is positive. Of course, the size of this positive effect on output depends on the changes in both the  $AD^S$  and the AS. Precisely, if the positive effect of shadow employment on the actual level of GDP is significant (AS falls considerably in the presence of the shadow economy),<sup>7</sup> then point C could potentially be a better situation than point A, since the cost of living is lower and purchasing power is higher.

Instead, in the long-run (compare points A and B in Figures 4 and 5), as suggested by empirical evidence (see, e.g. La Porta and Shleifer, 2008), in the presence of the shadow economy, the potential output is lower, since unproductive shadow firms use unskilled labour and adopt backward technology.

Therefore, an intertemporal socioeconomic dilemma emerges for policymakers:

(1) If the main goal of policymakers is economic growth, they should devote their greatest efforts to fighting against the shadow economy (tax evasion and corruption, first of all). Also,

<sup>7</sup> According to Pyle (1989), the supply-side effects generated by the existence of a shadow sector may be more substantial than the aggregate demand effects.

policymakers should adopt other economic policies that are compatible with this purpose, such as reducing both the tax burden and inefficient public expenditure, improving governance and the quality of political institutions, and strengthening the rule of law. Finally, policymakers should devote more economic resources to investing in both human capital and research and development. Of course, the achievement of these goals would require a lot of effort and a very long time.

(2) Instead, if the policymakers look especially at the present, the shadow economy could be tolerated to some extent. In some countries, this seems to happen. In the short-run, in fact, shadow employment could increase both employment and output.

This simple extension of the AD-AS model is capable of explaining the presence of the shadow economy in countries around the world, as well its persistence over time.

### Conclusion

Empirically, the shadow economy is a widespread phenomenon around the world that persists over time. Thus, the effects of a shadow economy deserve study at the basic level of introductory courses in economics. More precisely, the benefit of including the shadow economy in one of the most studied macroeconomic models at the undergraduate level, the AD-AS model, is twofold. From a theoretical point of view, the inclusion of tax evasion and shadow employment in the AD-AS model is straightforward, but the results are non-trivial. From an economic point of view, this paper makes this popular but “old” framework more alive. This simple extension of the AD-AS model is able to account for the presence and persistence of the shadow economy, despite its negative effects on economic growth.

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