SUPPLY LOCUS: A NOTE ON THE SUPPLY DECISIONS OF MONOPOLISTIC FIRMS

John Roufagalas[1]

Abstract

This note introduces the concept of the supply locus to describe the systematic supply responses of monopolistic firms to changes in demand. While in general the supply locus is not unique, it is shown that it is unique in the empirically interesting cases of a) a linear demand that shifts in a parallel fashion, and b) a constant elasticity exponential demand that shifts in a way that preserves its elasticity.

Key Words: Supply Locus, monopolistic competition, monopoly.

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Introduction

The Marshallian demand and supply model has been the workhorse of economics for more than a century now. While the demand side of the model has received considerable attention over the years, the supply side is problematic. It is generally claimed that the individual supply curve (and consequently the market supply curve) does not exist when the firm is not a price taker, *i.e.* when the firm does not face a horizontal demand curve, or equivalently, when the firm is not perfectly competitive.[2] Since perfect competition is an ideal state, rather rare in reality, then in theory, the supply curve should also be rather rare. Nevertheless, considerable empirical effort has been devoted to estimate supply curves and supply elasticities.[3] It seems a waste of intellectual energy to attempt to estimate something that theoretically (and presumably in reality) rarely exists.

This note claims that when a firm is not a price taker, *i.e.* it faces a downward sloping demand curve and determines its optimal output by equating marginal cost to marginal revenue, it responds systematically to changes in demand. These systematic responses can be used to derive

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[1]   Professor of Economics, Division of Economics and Finance, Troy University Montgomery, P.O. Drawer 4419, Montgomery, AL 36103.

[2]   For example, the leading Principles of Economics textbook these days, Mankiw (2015), devotes a separate box in page 308 to the claim that "…Monopoly Does Not Have a Supply Curve."

[3]   For example, Askari and Cummings (1977) report hundreds of supply elasticity estimates for agricultural products. Some of these estimates refer to categories like: "vegetables" or "soft fruits" that definitely do not satisfy the "uniform product" assumption of perfect competition. Traesupap, Matsuda and Shima (1999) study the demand and supply for Japanese shrimp defined as 7 different types, including lobsters, that is not uniform as perfect competition requires. Kenny (1999) estimates supply and demand for housing, even though it is difficult to argue that housing is a perfectly competitive market.

a "supply locus" that can take the place of the missing supply curve. Thus, the empirical work estimating supply curves, as well as public policy using these estimates, are actually estimating "supply loci." It will be shown that the position and slope (and hence, elasticity) of the supply locus depends not only on the marginal cost determinants that affect the traditional supply curves, but also on demand characteristics. That these demand characteristics are ignored in the empirical work implies that empirical estimates of supply slopes and elasticities may be biased due to missing variables.[4]

In what follows a quick reference is made to the argument for the non-existence of the supply curve when the firm has market power. Then, the "supply locus" for a monopolistic firm is derived.[5]

The Non-existence of the Supply Curve for Non-perfectly Competitive Firms

According to Pindyck and Rubinfeld (2013):

"A monopolistic market has no supply curve. In other words, there is no one-to-one relationship between price and quantity produced. The reason is that the monopolist's output decision depends not only on marginal cost, but also on the shape of the demand curve. As a result, shifts in the demand do not trace out a series of prices and quantities as happens with a competitive supply curve. Instead, shifts in demand can lead to changes in price with no change in output, changes in output with no change in price, or changes in both." (365)

Thus it appears that the non-existence of monopolistic supply is based on a non-uniqueness argument. That the Pindyck and Rubinfeld argument does not hold for "parallel shifts" of a linear demand and for constant elasticity demand curves is shown below. In other words, there is a one-to-one relationship between price and quantity supplies in two cases: 1) when the demand is linear and only shifts in a parallel way, *i.e.* it does not change slope; and 2) when the price elasticity of demand is constant and does not change as demand shifts. Non-uniqueness occurs when demand changes either location and slope both, or price elasticity.

Hence, if we restrict attention to either linear (for shifts not caused by the own price coefficient) or constant elasticity demand curves, then non-uniqueness does not apply.

Deriving a Supply Locus

Let the following be the Demand (D), Total Revenue (TR) and Marginal Revenue (MR) functions:

$$D = P(Q), \ P'(Q) < 0$$
$$TR = Q \cdot P(Q)$$

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[4]   While direct evidence for this claim requires extensive empirical testing, some indirect evidence can be gleaned from the existing literature. Just (1993) indicates that "…the estimated parameters from these simple models seem to be unstable over time so that forward-looking prediction and policy analysis are not well supported." It is argued here that the instability may partially derive from ignoring demand characteristics.

[5]   In the case of oligopoly, the Cournot model exogenously assumes a vertical supply relation (*i.e.* fixed quantities) and the Bertrand model exogenously assumes a vertical one (*i.e.* fixed prices). Models by Klemperer and Meyer (1989), Grossman (1981), Vives (2011) assume a "strategic supply curve" that allows both quantity and price adjustments, i.e. they assume a positively sloping supply.

$$MR = \left(\frac{\partial P}{\partial Q}\right) \cdot Q + P = P \cdot \left(1 + \frac{1}{e}\right), \quad where \ e = \frac{\partial P}{\partial Q} \cdot \frac{Q}{P}$$

Also let the following Total Cost (TC) and Marginal Cost (MC) functions be:

$$TC = C(Q)$$
$$MC = C'(Q) > 0$$

Primes denote derivatives and e stands for the price elasticity of demand. Standard monopolistic profit maximization, MC = MR, determines the optimal quantity Q* supplied by the monopolist:

$$MC = MR \ => C'(Q^*) = P(Q^*) \cdot (1 + \frac{1}{e})$$

Given the optimal quantity, the monopolistic price can be determined by adding the profit margin (P-MR) to the marginal cost (MC). This allows the derivation of the Supply Locus:

$$P(Q) = MC + (P(Q) - MR) = C'(Q) - \left(\frac{1}{e}\right)P(Q)$$

or

$$P(Q) = C'(Q)(\frac{e}{e + 1})$$

This is a version of the well-known "markup" equation. For a perfectly competitive firm, as the elasticity of demand is (minus) infinity, the elasticity term equals one and the supply locus coincides with the supply curve. For e < -1, the elasticity term is positive and larger than one, indicating that the supply locus will lie above the marginal cost function. For inelastic price elasticities of demand larger than or equal to minus one, there is no interior monopolistic profit maximization solution, i.e. corner solutions apply.

The supply locus equation makes clear that its shape and location depend not only on the factors that affect marginal costs (*i.e.* the traditional supply factors,) but also on the factors that affect the price elasticity of demand.

Obviously, when the elasticity of the demand is constant and does not change when the demand shifts, the Supply Locus is unique and its slope is proportional to that of marginal cost:

$$P'(Q) = C''(Q)(\frac{e}{e + 1})$$

Alternatively, when demand shifts result in changes in price elasticity, as when demand is linear, then the supply locus also shifts. Then it is possible for demand to intersect the supply locus at the same price or the same quantity as before the shift. That is, there is non-uniqueness.

Now consider the linear demand specification of the model.

A Linear-Quadratic Specification

The supply locus for a model with linear demand and a quadratic total cost function is derived below. This specification is interesting, because almost all textbooks draw a linear downward-sloping demand and a linear upward-sloping marginal cost. In addition, a large segment of the empirical work focuses on linear demand and supply curves.

Suppose the following demand, marginal revenue, total cost and marginal cost functions:

$$P = a + \left(\frac{b}{2}\right)Q, \quad a > 0; b < 0$$
$$MR = a + b\,Q$$
$$TC = c_0 + c_1 Q + \left(\frac{c_2}{2}\right)Q^2, \quad c_1, c_2 > 0$$
$$MC = c_1 + c_2 Q$$

Equating MC to MR, and subsequently using the demand function yields the following optimal price/quantity combination:

$$Q_M = \left(\frac{a - c_1}{c_2 - b}\right)$$

$$P_M = \left(\frac{2a(c_2 - b) + b\,(a - c_1)}{2(c_2 - b)}\right)$$

The margin P-MR equals $\left(-\frac{b}{2}\right)Q$. Hence the Supply Locus is:

$$P_M^{SL} = MC + (P - MR) = c_1 + \left(c_2 - \frac{b}{2}\right)Q$$

Inspection of the supply locus function reveals that a) if b=0, *i.e.* the demand is horizontal as in perfect competition, the supply locus coincides with the supply curve/marginal cost; b) the slope of the supply locus is the sum of the slopes of the marginal cost curve and the demand curve; and c) as long as demand shifts derive from changes in the demand intercept without changing its slope, the supply locus is unique and the equilibrium price-quantity combination will be at the intersection of the demand curve and the supply locus.

Figure 1
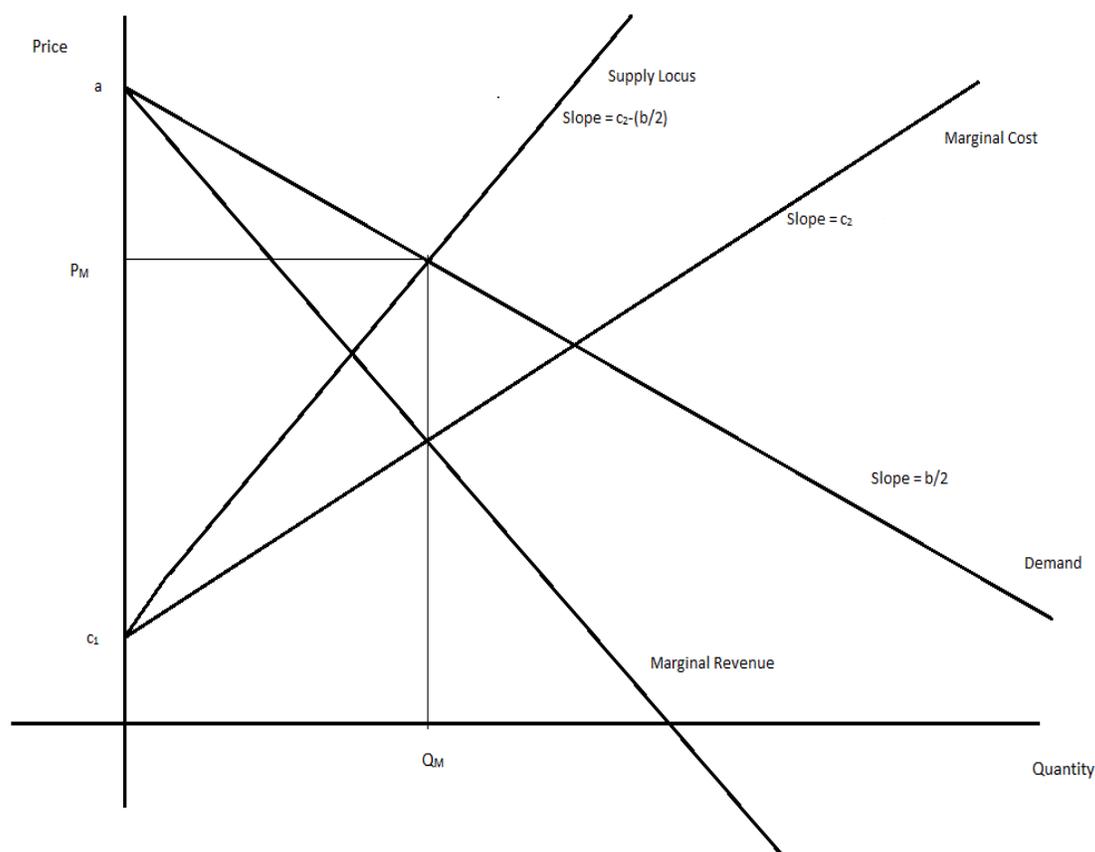
The significance of this result for empirical work is that observed market equilibria in non-perfectly competitive markets are, most likely, points at the intersection of the demand with the supply locus, not at the intersection of the demand with the marginal cost curve. Assuming that

the estimated system is linear with a stable own price coefficient on the demand side (*i.e.* a constant demand curve slope), it is obvious that on the supply side, the correct estimation is that of the supply locus and not of the marginal cost/supply curve. Ignoring the effect of the slope of the demand on the supply locus leads to systematic bias on the estimated supply coefficients (missing variables bias.)

Incorporating the Supply Locus in Economics textbooks

The central goal of this discussion has been to demonstrate that, under some conditions, a version of the supply and demand model can be used in non-perfectly competitive markets. In other words, the model's insights about price determination and price changes apply in a much wider range of markets. Given that short run analysis is identical for a pure monopoly and a monopolistically competitive firm, and given that pure, unregulated, monopolies are rather rare, the model is most relevant to monopolistically competitive market structures.[6]

There are various ways to refer to the supply locus in a textbook. In a Principles textbook, a short section or a box could be added to the supply and demand chapter, claiming that under some conditions, *i.e.* linear demand and parallel shifts, the demand and supply model (with an adjustment on the supply side) has much more general applicability than the perfect competition assumptions suggest. Another option is to derive the supply locus in the monopolistic competition or the monopoly chapter, adding the claim that while the monopolistic firm has no supply curve, it does have a supply locus which, under some circumstances, is unique.

In an Intermediate Microeconomics textbook, a geometric and/or algebraic derivation can be added in the Monopoly or Monopolistic Competition chapter.[7]

Conclusion

This note derived the optimal supply responses of a monopolistic firm and dubbed them a "supply locus." The analysis showed that when demand is linear or exponential and, over time, demand shifts preserve the demand slope or elasticity respectively, the supply locus is unique. If,

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[6]   Shepherd (1982) presents data that show what he calls "Effective Competition" (made up of Perfect Competition, Monopolistic Competition and Loose Oligopoly) accounted for about 76.7% of the National Income, while Pure Monopoly accounted for 2.5% of National Income in 1980.

[7]   If added in the Monopolistic Competition chapter it offers the intriguing possibility of providing a connection to product differentiation and hedonic estimation. Combining the demand for product attributes introduced by Lancaster (1966) with the hedonic estimation of the underlying prices for these attributes introduced by Rosen (1974), one could show how the demand and supply locus for a particular product could be disaggregated to demands and supply loci for individual attributes. Adding all demands and supply loci for each individual attribute (*i.e.* over all the differentiated products in the market) one can derive the market for that attribute and determine its price (which hedonic models try to estimate). Using these market prices and the composition of each product, one can then derive the equilibrium price for each differentiated product, which should be consistent with the price obtained at the intersection of the demand and supply locus for that product.

however, demand shifts entail simultaneous intercept and slope changes in the linear case, or elasticity changes in the exponential case, then the supply locus is non-unique.

In empirical work using linear or logarithmic specifications, where a stable slope or elasticity parameter is estimated, the corresponding supply estimation suffers no major econometric problems.[8] The estimated relationship is simply the supply locus instead of the supply/marginal cost function.

Problems arise when non-linear, non-logarithmic functions are used to estimate supply and demand systems. In such cases the supply locus is not unique and the supply equation should include demand elasticity determinants to be fully identified. Another problem is that it is not appropriate to use estimated supply locus parameters to make inferences about the parameters of the marginal cost function in the absence of the adjustments that follow from the above analysis.

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[8]   Interestingly, Just (1993, section 4.6) argues that log-linear (Cobb-Douglas) supply estimations are sometimes more reasonable that more general functional forms such as the translog.