

# Scholastic Aptitude and Studying For an Exam: Input Substitution in Grade Production

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## Abstract

The score on the first exam in a principles of macroeconomics course is estimated as a function of time spent studying and academic background (as measured by grade point average). Results show that academic background is relatively more important than studying, though both inputs are subject to diminishing returns. Exam score is also subject to decreasing returns to scale.

## 1 Introduction

ECON202, Principles of Macroeconomics, is a three-hour course offered by the College of Business at Tennessee Technological University. The course is required for all students seeking a degree in business (it is part of the “core curriculum”), and is taken as an elective by a large number of other majors. There are no prerequisites for the course, though it is numbered as the second in a two-course sequence of principles of economics. This, and the fact that the course was offered in the Spring semester, means that all but one of the students sampled had taken the Principles of Microeconomics in a previous semester (and that one student was concurrently taking ECON201).

The course is taught by almost all members of the economics faculty, with two sections offered in the fall semester, six in the spring, and one in the summer term. Principles classes had an average enrollment of 54 in the Spring of 2000, which is about twice as large as the average class size for the university. One section had an enrollment of 91, and the others 49-53.

Because of the relatively larger class sizes, and the lack of instructional technology in the classroom, the teaching method is almost exclusively lecture. Exams are all multiple choice.

## 2 Sample

A questionnaire was given to each student attending class in two sections of ECON202 on March 9, 2000. The date is significant because it was two days after the “Super Tuesday” primary/caucus nomination day for U.S. President, and five days prior to the Tennessee Presidential primary. Although the main purpose of the study was to determine the impact of study hours and academic disposition on grades on the first exam in ECON202, additional information was collected on demographics and voting intentions.

Students were told that responses to the questionnaire would remain anonymous, and for that reason, some potential questions that would reveal the student's identity could not be asked. A total of 83 questionnaires were returned.

At the time the questionnaire was given, the students had taken only one exam, about two weeks prior. The material covered on the first exam was introductory material, and general concepts of economic growth. At the time of the questionnaire, the course had not yet begun to cover short-run economic fluctuations (which is to say, they had not been introduced to Keynesian economics).

## 3 Descriptive statistics

### 3.1 Demographics

Of the 83 students who turned in the survey, 61 percent were male, and 39 percent were female. Approximately 24 percent were over the age of 21, which is beyond the age of the traditional college student. Thirty percent lived on

campus, and 58 percent lived off-campus but within a 30 minute commute. Twelve percent commuted more than 30 minutes (one way) to the university.

### 3.2 Awareness of events

When asked whether the student felt he or she was “well-informed”, only 16 percent indicated they were “very well” informed, while 40 percent considered themselves “uninformed”. They were generally correct in their perceptions of themselves. Two questions were asked that provided some indication of just how well informed students were: the identification of John M. Keynes and Patrick J. Buchanan.

Regarding Keynes, only 53 percent of the students correctly identified him as an economist. While this is better than chance, and given that the students had yet covered Keynesian economics, it is still disturbing that only half could identify the person who was arguably the most influential economist of the 20th century. As Table 1 reveals, as the perception of how well informed the students felt they were increased, the proportion of those who correctly answered the question increased as well.

Table 1: “Who is John M. Keynes?” and Informed				
Answer to question was	“how informed are you?”			
	uninformed	informed	very informed	
Correct	14	20	8	42
Incorrect	18	15	4	37
	32	35	12	79

The students redeemed themselves in the eyes of their instructor by actually being *less* informed about the identity of Patrick J. Buchanan. The exact same students who classified themselves as uninformed or very informed were incorrect in their identity of Buchanan as they were Keynes. But three of those who were able to correctly identify Keynes were incorrect in their identification of Buchanan.

Table 2: “Who is Patrick J. Buchanan?” and Informed				
Answer to question was	“how informed are you?”			
	uninformed	informed	very informed	
Correct	14	17	8	42
Incorrect	18	18	4	37
	32	35	12	79

The reason that these students are not well-informed is that they do not read news media. As the table below indicates, there is a strong relationship between how often students read news in newspapers and magazines and how well informed they feel they are.

Table 3: Reading News and Informed				
“how often do you read news?”	“how informed are you?”			
	uninformed	informed	very informed	
< 1 time per week	16	6	2	24
2-5 times per week	15	29	6	50
> 5 times per week	2	1	5	8
	33	36	13	82

It isn't just news the students don't read, either. When asked how many books they had read in the past year that were not required for a class, 78 percent responded that they had read fewer than 5 books. Even more disturbing: 23 percent had read no books at all in the past year.

## 4 Determinants of exam scores

The average grade on the first examination was 78.05 (a high C), with a standard deviation of 10.81. Faculty are often interested in what determinants affect course performance, and have offered different opinions. Some feel that students have too many demands on their time, and/or waste their time. It is often observed that more and more of our students are working, and spend a greater number of hours during the semester in their jobs. Students who spend a greater amount of time commuting to campus may be at a disadvantage to others who live on campus. And watching excessive amounts of television is often blamed for poor exam scores. Some faculty feel that students are not “well-read”, or are poorly informed.

None of these things had a discernible effect on the performance on the first exam. Only two things had a statistically significant effect: time spent studying, and grade point average. Students answered a question on the questionnaire about the number of hours they studied for the exam. They were also asked what their cumulative grade point average was prior to the beginning of the semester. There are very few studies of the determinants of college GPA<sup>1</sup>. Recent work by Betts and Morell [1] shows that college GPA depends on

1. the degree program in which students are enrolled (some disciplines are “harder” than others),
2. the student's family background (eg, family income and race),
3. high school resources prior to enrolling in university (teachers' experience was the most important), and
4. the demographic environment in which the student attended high school (levels of education or income in the community).

For the purposes of this study, GPA serves as a proxy for “academic background” or “scholastic aptitude”.

#### 4.1 Theoretical model<sup>2</sup>

Students can be viewed as “producing” exam scores in much the same way that economic theory views firms producing output. In microeconomics, firms produce output by using combinations of resources (or “inputs”). The *production function* relates the inputs to the firm's output. In the same manner, students combine study hours and grade point average to produce exam scores.

The “production function” that relates exam score to the use of these two resources is

$$Y = F(K, L) \quad (1)$$

where  $Y$  is the score on the first exam,  $K$  is grade point average, and  $L$  is hours spent studying for the exam.

The symbol  $F$  in equation (1) represents the functional form. A popular functional form for the empirical estimation of production functions is the *Cobb-Douglas production function*, which can be specified as

$$Y = AK^\alpha L^\beta, \quad (2)$$

where  $A$ ,  $\alpha$ , and  $\beta$  are parameters to be statistically estimated.

##### 4.1.1 Marginal products

An important consideration in the discussion of the production function is the impact on output of changing one resource while holding the other resource constant. In the microeconomic theory of the firm, this is referred to as the *marginal product* of the input. In the current model, the marginal product of studying is

$$\frac{\partial Y}{\partial L} = \beta AK^\alpha L^{\beta-1} \quad (3)$$

and the marginal product of GPA is

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<sup>1</sup> But there are numerous studies concluding that GPA is an important determinant of future income.

<sup>2</sup> Consult any good intermediate level microeconomics text, such as Nichols [3] for enhanced discussion of the theory in this section.

$$\frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1}L^{\beta} \quad (4)$$

#### 4.1.2 Input substitution

It might be possible for a student to compensate for a poor academic background (a lower GPA) by studying more. That is, a student with a lower scholastic aptitude could score the same as a student with a higher aptitude by studying more. Alternatively, one student could have studied much less for the exam than another student, and achieved the same exam score, due to his or her superior academic background. In terms of microeconomic theory, one input can be substituted for another, holding output (or, in this case, grade) constant. The *marginal rate of technical substitution* shows the rate at which studying can be substituted for GPA, holding the exam score constant.

To determine the marginal rate of technical substitution from the production function given in equation (2), first take the total differential,

$$dY = \frac{\partial Y}{\partial L} \cdot dL + \frac{\partial Y}{\partial K} \cdot dK, \quad (5)$$

which tells us how small changes in  $L$  and  $K$  affect exam score. Holding the exam score constant means that  $dY = 0$ , so

$$\frac{\partial Y}{\partial L} \cdot dL = -\frac{\partial Y}{\partial K} \cdot dK. \quad (6)$$

Equation (6) says that for a given exam score (call it  $Y_0$ ), any increase in the score from increasing study time is exactly balanced by the loss in exam score from a suitable decrease in GPA. Rearranging the terms in (6) gives the common expression for the marginal rate of technical substitution:

$$\frac{dK}{dL}_{Y=Y_0} = \frac{\partial Y/\partial L}{\partial Y/\partial K} = \frac{\text{marginal product of } L}{\text{marginal product of } K}; \quad (7)$$

that is, the marginal rate of technical substitution is equal to the ratio of marginal products.

#### 4.1.3 Factor shares

In the theory of the firm, we sometimes measure how much of the output can be attributed to the various inputs; that is, we measure labor and capital's "share" of the output (which in purely competitive markets would equal the factor payments)<sup>3</sup>. In the present study, we can measure the relative importance of studying and scholastic aptitude. The share of the exam score attributable to studying is

$$\frac{\partial Y/\partial L}{Y} = \frac{\beta Y}{Y} = \beta \quad (8)$$

and GPA's share is

$$\frac{\partial Y/\partial K}{Y} = \frac{\alpha Y}{Y} = \alpha. \quad (9)$$

This is quite convenient: the measure of the relative importance of each of the inputs is the exponent of the input in the production function.

#### 4.1.4 Returns to scale

Instead of examining the impact of a change in one input in isolation on exam score, we could measure the impact of a proportionate change in both inputs. This is the concept of *returns to scale*. A production function exhibits *constant*

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<sup>3</sup> In macroeconomics, we speak of labor and capital shares of *national income*. The production function is one of aggregate output instead of a firm.

returns to scale if an increase of an equal percentage in all inputs causes an increase in output of the same percentage; ie,

$$y = F(zK, zL), \forall z > 0.$$

If a proportionate increase in inputs increases output less than proportionately,

$$zY < F(zK, zL),$$

the function exhibits diminishing returns to scale. And if output increases more than proportionately,

$$zY > F(zK, zL),$$

there are increasing returns to scale. For a Cobb-Douglas production function like the one specified in this study,

$$\begin{aligned} F(zK, zL) &= A(zK)^\alpha (zL)^\beta \\ &= Az^{\alpha+\beta} K^\alpha L^\beta \\ &= z^{\alpha+\beta} Y. \end{aligned} \tag{10}$$

Thus, if  $\alpha + \beta = 1$ , the function exhibits constant returns to scale. If  $\alpha + \beta > 1$ , the function exhibits increasing returns to scale, and if  $\alpha + \beta < 1$ , decreasing returns to scale.

#### 4.1.5 Short-run vs. long-run

The production function of equation (2) assumes that both  $L$  and  $K$  can be varied. Sometimes when discussing the production function for a firm, the time horizon is divided into the short-run and long-run. In the short-run, some inputs may be fixed (for example, the plant) and some may be variable (labor). Output can be increased only by increasing labor. The change in output resulting from the increase in labor is the marginal product of labor. In the long-run, the firm has the opportunity to expand the plant size (or build another, larger plant), so that all resources are variable, and none are fixed.

Likewise, we can examine exam scores in the same time horizons. For the given class of college students, their academic background (measured as GPA) is fixed. The only way they can increase the grade on a single exam is to study. But if we were to extend the time horizon, we could examine the effect of providing a better academic background to a student, so that all inputs are variable.

## 4.2 Empirical estimates

In this section, empirical estimates of the production function are provided. The function as expressed in equation (2) is nonlinear and computationally difficult to estimate. Taking the natural logarithm of the function,

$$\ln Y = \ln A + \alpha \ln K + \beta \ln L \tag{11}$$

makes a linear transformation that can be estimated with ordinary least squares regression<sup>4</sup>. Table 4 provides the results of the estimation of equation (11).

Variable	Parameter estimate	standard error	t-statistic	prob >  t
A	3.8590	0.081855	47.145	0.0001
Ln L	0.0501	0.017735	2.827	0.0061
Ln K	0.4163	0.071809	5.797	0.0001
R <sup>2</sup> =0.3347, F=18.366, n=76				

The regression estimate is, then,

$$\ln Y = \ln 3.859 + 0.4163 \ln K + 0.0501 \ln L.$$

These estimates have the following interpretations in light of the discussion in section 4.1:

1. Students use studying and academic background to “produce” exam scores according to the function

<sup>4</sup> This is a common estimation procedure that is detailed in any good introductory econometrics text, such as [2].

$$Y = 47.4179K^{.4163}L^{.0501}.$$

2. Each additional hour of studying increases the exam score by

$$2.3765K^{.4163}L^{-.9499}.$$

This is the marginal product of studying. Notice that its value depends on the values of<sup>5</sup>  $K$  and  $L$ . At the mean values of  $K$  and  $L$  (this might be regarded as the “average” student), an additional hour of studying raised the exam score by only 1.0948.

(a) Exam score is subject to diminishing returns with respect to studying. If we consider the student with the average GPA of 2.925, the marginal product of studying is

$$3.7138L^{-.9499}.$$

This means that the first hour of study increased the exam score by 3.7138, but the second hour added only 1.9225. A third hour of study increased the grade by 1.308, while the fourth added slightly less than one point to the exam score.

(b) An hour of study is more effective in raising the exam score the better the student's academic background. For a student with a grade point average of 2.0, the first hour of studying increased the exam score by 3.1702. For a student with a GPA of 3.0, the first hour of studying raised the score by 3.75. An alternative way of looking at this is that a student with a poor academic background must study more to achieve the same exam score.

3. The marginal product of  $K$  (how much a one unit increase in GPA changes exam score) is

$$19.7401K^{-.5837}L^{0.0501}.$$

At the mean  $K$  and  $L$ , the marginal product of  $K$  is 11.2538. Notice that exam score is also subject to diminishing returns with respect to GPA.

4. The marginal rate of technical substitution (MRTS) can be calculated as the ratio of marginal products, as demonstrated in equation (7). At the mean values of  $K$  and  $L$ , the MRTS is 0.0973. This implies that for the average student, an hour reduction in studying can be exchanged for a 0.0973 increase in GPA, and the exam score will remain the same.

As is typical in production processes, the MRTS is different at different levels of  $K$  and  $L$ . Consider a hypothetical student who studied quite a bit for the exam (say, 5 hours), but had a poor academic background (a GPA of 2.0). The MRTS for this student would be 0.0481. This means that it becomes increasingly difficult to trade off an hour of studying for GPA the fewer hours that are studied and the greater the GPA. That is, MRTS decreases with increases in study time.

5. It is easy to make too much of the estimate that, for the average student, the marginal product of  $L$  is 1.0948 and the marginal product of  $K$  is 11.2538, since the size of the coefficients of the regression equation depends on the measurement scales of the independent variables. For example, if GPA had been measured in hundredths of units, the marginal product of  $K$  would have been approximately 0.1. A better measure of the relative importance of  $L$  and  $K$  is the “factor shares” as expressed in equations (8) and (9). Given the estimation equation in (11), the values of  $\alpha$  and  $\beta$  do not depend on the scale of measurement<sup>6</sup>. Accordingly, the share of the exam score attributable to studying is

<sup>5</sup> More accurately, on the ratio of  $K$  to  $L$ .

<sup>6</sup> This is because taking the natural logarithm is, in the limit, equivalent to measuring percentage changes.

only 0.0501, while the share attributable to GPA is 0.4163. Hence, academic background is much more “important” to the success on an individual exam than studying.

6. It may be somewhat confusing that the “shares” of exam scores attributable to  $K$  and  $L$  do not sum to one. The reason, as derived in equation (10), is that since  $\alpha + \beta < 1$ , exam scores are subject to decreasing returns to scale. A ten percent increase in *both* study time and GPA would increase the exam score by only about 4.5 percent.

## 5 Conclusion

Studying improves a student's exam score, but academic background is relatively more important. That is, academic background, as measured by prior cumulative grade point average, adds more to exam score than studying. This highlights the importance of pre-university schooling, and diminishes the importance of the individual university professor (at least, the professor of economics). The evidence from this study can be added to the list of arguments calling for more resources to be devoted to secondary education.

Of course, once a student enrolls at university, academic background is fixed. The only way to increase the score on any individual exam is to increase study time. However, for students with good academic backgrounds, study is more efficient, so that less time must be spent studying to achieve the same exam score than must be spent by students with lower scholastic aptitude.

This study is not without problems. There are likely other determinants of exam score, such as class attendance, that were not included since the survey would not have been anonymous. The survey question “How many hours did you study for this exam” might mean different things to different students. For example, does this mean just the time spent the night before the exam, or does it include the time spent the prior week reading the text, or the time spent in class. A refinement in the survey instrument is called for. The most significant problem is the obvious endogeneity of the score on individual exams collectively determining GPA, which the methodology ignores. These are left, as usual, to future research.

## References

- [1] Julian R. Betts and Darlene Morell. “The Determinants of Undergraduate Grade Point Average”, *Journal of Human Resources*, 34(2):268-293, 1998.
- [2] G. S. Maddala. *Introduction to Econometrics*, MacMillan, 1988.
- [3] Walter Nicholson. *Intermediate Microeconomics and Its Applications*. Harcourt, 8<sup>th</sup> edition, 2000.