



---

## Emerging Scholar

### “Productive Struggle” as an Effective Strategy in Elementary Math Classrooms

Sara Daily<sup>a</sup>

<sup>a</sup>Middle Tennessee State University

Sara Daily graduated from Middle Tennessee State University with her Bachelor's in Film and Video Production in 2016. She is currently earning her master's degree at Middle Tennessee State University in Curriculum and Instruction with an emphasis on Elementary Education in the Initial Licensure Specialization. Her research interests focus on improving education for all elementary students.

## Abstract

*Productive Struggle* refers to a strategy that gives students an opportunity to increase their background mathematical knowledge. Productive Struggle helps students connect key concepts, determine how and where an error occurs, and supports students in figuring out how to use their own thinking and reasoning skills to correct an error. Teaching without utilizing Productive Struggle instruction can lead to students being reluctant to work towards developing a deeper understanding of math concepts. Without a deeper understanding of mathematical concepts, students, who often do not perceive themselves as competent in math, may continue to find math difficult for them in the future.

## Introduction

Productive Struggle (Murdoch et al., 2020) is an instructional strategy designed for teachers to help students use their own thinking and reasoning skills to solve math problems. The process helps students develop an improved understanding of conceptual math as well as to gain confidence in their abilities to solve problems.

Productive Struggle instruction is not a concept that is easily implemented. It requires teachers to know the material well enough to determine where a student may go wrong in the process and then, know how to guide them to the correct solution without directly providing the answer. Productive Struggle, as a part of instruction, enables students to develop confidence in their abilities, become more willing to take risks, and grow in their understanding of the content.

When students are building a new skill, it is not always comfortable for them. In addition, allowing students to struggle is typically not in most teachers' comfort zone, which begs the question: What is to be gained from this uncomfortable and time-consuming method?

## **Productive Struggle Instruction**

Productive Struggle, as explained by Murdoch et al. (2020), is recognized both in policies and research as a signal of quality education. They believe Productive Struggle is an important and necessary strategy for teaching mathematics for conceptual understanding. Instruction in Productive Struggle can help students by teaching them how to problem-solve effectively. These skills will help students, not only in their current classroom, but throughout their educational time as well.

Traditionally, a common method of instruction in mathematics involves teaching students a formula or pattern to follow. These instructional methods provide students with the correct answer in a timely manner, but unfortunately, do not teach students how to independently solve problems. This leads to other issues in both their future math classes and in real-life situations.

Mathematics teaches students the important skills of how to problem solve and adapt in a variety of situations, not just get right answers. By only teaching formulas and not problem solving, teachers deny their students the opportunity to perceive themselves as competent at math; this perception undermines students' potential to become a problem solver.

Math problems are often tasks without clear solution paths. Effective teachers can provide their students with more than just a formula; rather, they can highlight the possibility of how students can solve problems by themselves in a variety of ways. The instructional method used to teach problem solving and help aid students in becoming more confident in their mathematic abilities is called Productive Struggle. Productive Struggle is an effective method for enhancing the process of mathematics teaching for the benefit of students.

## **Productive Struggle Goals**

Productive Struggle instruction goals include students learning to think deeply, gaining understanding, and developing independent problem-solving skills (Amidon et al., 2020; Baker et al., 2020; Granberg, 2016). While students who develop their own methods may encounter more errors and take longer to solve the problems, they often score more positively on tests than students who use memorized procedures. Even if the students who created their own methods are incorrect, they are more likely to adapt their methods than their classmates who used memorized information (Granberg, 2016). By creating their own methods, students can create new pathways and develop connections to concepts previously taught in mathematics. Without assistance from teachers, this is much more difficult for students to accomplish if all they have previously learned to do is memorize how to solve a problem.

As teachers, one of the most limited resources is time. Spending time doing something deemed unproductive is not a common practice. It is more likely that teachers will evaluate struggling students as doing something unproductive and provide those students with steps in how to solve the problem rather than allowing the students to develop the techniques themselves. There is a possibility when teaching problem-solving, that teachers will reduce it to following an algorithm rather than allowing the students to reason it out for themselves (Brousseau & Gibel, 2005).

Another problem may arise when teachers become unwilling to allow their students to fail, so they make the problem easier to solve, taking away the chance for the students to develop problem-solving skills (Brousseau & Gibel, 2005). Struggle takes time and is often not seen as worth the risk to a teacher not familiar with the benefits of Productive Struggle.

However, teachers who use Productive Struggle are taking a risk. First, implementation of Productive Struggle requires teachers to truly know their content. Thorough knowledge of the subject matter increases teachers' effectiveness in challenging and encouraging their students to engage in Productive Struggle (Murdoch et. al., 2020).

Second, learners possess diverse background knowledge and require different strategies to challenge them. Because of this, unpredictable behaviors often occur as they struggle during learning events. Teaching Productive Struggle not only takes preparation and an in-depth knowledge of the content but also understanding of the individual students' needs in order to be implemented effectively.

Most research describing Productive Struggle regards secondary students struggling in mathematics (Baker et al., 2020; Granberg, 2016; Warshauer, 2015; Zeybek, 2016). Research describing implementation of Productive Struggle in elementary schools is limited. However, this approach is complementary to Constance Kamii's (1982, 2000) constructivist understanding of how young children (ages birth to eight) develop logico-mathematical knowledge. Kamii (1982) asserts, "Relationships are created by the child from within, and not taught by someone else from the outside" (p. 29). She advises teachers to create an environment that encourages children to think and make their own decisions. Kamii (1982) explains that the teacher's role is not simply correcting children's answers, but rather figuring out how the child made the error, and then guiding the child's "process of reasoning which is far better than correcting the answer" (p. 41). Similar to Productive Struggle, Kamii's (2000) approach supports children developing their thinking skills, rather than memorizing rules, as this will enhance their understanding of math concepts as well as build their confidence.

Lev Vygotsky's theory, zone of proximal development, can be found in several Productive Struggle studies. Struggle involves students doing scaffolded tasks that are within the student's understanding (Betts & Rosenberg, 2016; Vazquez et. al., 2020). Scaffolding allows the students to take steps towards the desired goal at a reasonable pace. These scaffolded tasks can lead to better memory of the material, a deeper understanding, and a chance to create more solutions to the same problem (Vazquez et. al., 2020). The key to Vygotsky's theory is for teachers to maintain high expectations of their students. In a study by Ewing et al. (2019), English language learners were thought to not have mastered English and were provided with problems outside their zone of proximal development. These expectations only hurt the students and the teachers. By providing problems within the zone of proximal development, students can develop their mathematic skills without becoming overwhelmed.

Brousseau's theory of didactical situations in mathematics has been implemented in Productive Struggle research as well. Granberg (2016) states that mathematics without struggle involves fast ways that will always lead to the correct answers. While this is an ideal output and useful in saving time, it does not prepare students for the future. Teachers wish for their students to

succeed. Some teachers find that when leading their students, they break down the problems until they no longer require critical thinking (Brousseau & Gibel, 2005). Effective teachers do not simply provide their students with procedures but allow the students to solve problems on their own and develop the skills necessary for all subjects.

The earlier students are supported in working through difficult problems, the better prepared these students will be throughout their education. Problem-solving skills are not limited to mathematics. Students who are supported in developing their own methods will likely be more successful in many other school subjects. Teachers may not initially acknowledge the time spent teaching Productive Struggle as worthwhile, but their students' test scores will reflect the benefits. More importantly, the students will become confident problem-solvers, an important life skill.

### **Benefits for Students**

Researchers consider Productive Struggle as a crucial and natural part of the learning process in mathematics (Murdoch et al., 2020; Russo et al., 2021). Math problems are not meant to be similar. They are intended to be implemented in a variety of situations and designed to prepare students for life after the classroom. Through intellectual struggle, students learn from their own mistakes. Instruction in Productive Struggle encourages students to implement this knowledge in other tasks and improves their ability to be self-directed (Lemley et al., 2019).

Productive Struggle provides students with opportunities to thoroughly study difficult problems in order to determine similarities between them. This helps students to develop a deeper understanding of mathematics (Ewing et al., 2019; Lemley et al., 2019; Russo et al., 2021). Allowing students to correct their mistakes is another means to build mathematical understanding. This knowledge cannot be achieved unless students can determine how their mistakes were made, or if their teachers specifically guide them to where they erred in their understanding.

The idea of struggle can be defined as students attempting to understand something in mathematics that is not clearly discernible at first glance (O'Dell, 2018; Warshauer, 2015; Warshauer et al., 2021). The problems meant for Productive Struggle are difficult, but not impossible. Problems requiring students to think about the process of finding solutions provide challenge as they use their thinking and reasoning skills. Importantly, if they can immediately use memorized information, the problem does not promote Productive Struggle. Struggle is only productive when teachers implement it correctly, and students are able to gain understanding and problem-solving skills from the effort. Herein, lies the benefit for students engaging in Productive Struggle.

Productive struggle builds students' understanding of mathematical concepts (Warshauer et al., 2021). The more students do on their own will promote future mathematics learning. By productively struggling, a more thorough knowledge of the topic is developed. Students remember concepts longer and better than if they had only been taught the steps to solve by their teachers (Vazquez et al., 2020; Warshauer et al., 2021).

Productive Struggle indicates where students are lacking in their knowledge (Amidon et al., 2020; Granberg, 2016; Murdoch et al., 2020; Vazquez et al., 2020). If a teacher walks the student through a problem, there is no critical thinking involved. The students cannot determine if they know how to do all the steps on their own. By allowing the students to figure out the problems for themselves, they can discern the differences between their current knowledge and what they are trying to understand (Granberg, 2016). These students will be able to make their own connections to other topics, which will help them with encountering future concepts and reviewing learned material.

Students feel positive emotions when they productively struggle. When students are successful in their struggles, they feel pride (O'Dell, 2018). There is something special about being able to solve something without any aid from others. The more often students are afforded opportunities to feel prideful in their work, the more likely they are to see mathematics in a positive light. Students were also noted to feel joy when they finished a problem (O'Dell, 2018). When a teacher can link happiness together with mathematics, students will benefit. When students can fail and still want to continue dealing with difficult problems, Productive Struggle instruction is successful (Livy et al., 2018).

### **How to Teach Productive Struggle**

Effective teachers instruct students in how to think like problem solvers. Through guiding and questioning, students become encouraged to determine exactly what is necessary for solving math problems. In general, a student's response to Productive Struggle is to practice, ask for assistance, or persist in struggle (Warshauer et al., 2021). When a student seeks assistance, it will usually come from a lack of understanding. The goal for teachers in this situation is not to tell the students specifically how to solve the problem, giving them steps and protocols, but instead, to scaffold and support them with guidance by providing students with strategies to help them to create their own methods and be able to solve the problems themselves in the future (Ewing et al., 2019; Warshauer, 2015).

There are several different strategies that can be used to teach Productive Struggle. For example, during instruction, the teacher talks the students through what they already know and provides them with questions requiring deep thinking. Providing questions for students encourages them to take the time to make sense of the problem they are trying to solve (Ewing et al., 2019). Unfortunately, there is not a set list of questions to ask. Productive Struggle requires teachers to adapt to each individual student and instruction varies with each type of problem (Lemley et al., 2019; Murdoch et al., 2020).

Another strategy used to teach Productive Struggle involves teachers inspiring their students to reflect upon their own work and become capable of explaining how they arrived at a particular answer. The ability to explain their own thinking enables students to develop their problem-solving skills. When students explain or show their reasoning behind how they arrived at their answer, the teacher helps guide the students with the knowledge of where they erred. In this way, students improve understanding and develop those skills for the next problem (Baker et al., 2020; Betts & Rosenberg, 2016; Murdoch et al., 2020).

Another strategy to implement Productive Struggle instruction entails teachers initially providing students with problems in a new concept, and then providing instruction *after* the students have had time to explore (Vazquez et al., 2020). This strategy builds on student understanding of the material because the students can develop their own reasoning on how to solve the problem. They determine the elements of the problem and create connections with problems they solved in the past with similar elements.

A common problem when teaching Productive Struggle is to reduce the cognitive load. Teachers are accustomed to achievement that seems to be effortless (Livy et al., 2018). Most teachers wish to help a student who seems to be struggling. Not doing so appears neglectful, but this is not always the case. Students require adequate time to develop persistence to be able to complete the tasks themselves. When teachers help students solve problems, they diminish students' chances to build critical thinking skills, support their dependence on others when confronted with challenges, and extend students' struggles in the future.

The key to teaching Productive Struggle is to provide students with the tools to independently solve the problems (Warshauer et al., 2021). Similar to reading instruction where students learn to decode words themselves, teachers can help their students develop the ability to create a list of approaches to solving a problem. The more the students can do by themselves, the more effective their learning.

Effective teachers purposefully plan the time to create an environment where students are able to develop a deeper understanding of the topics. They encourage learning through process and building knowledge rather than attaining correct answers. While these teachers maintain high expectations of their students and their abilities, they prepare for and support Productive Struggle; when they do not, they deny students opportunities for growth and understanding (Ewing et al., 2019).

### **Productive Struggle in the Classroom**

In Productive Struggle instruction, the goal is not for students to consistently and correctly solve the problems. Effective teachers shift their understanding of success to create opportunities for their students to build knowledge, not just show correct answers (Vazquez et al., 2020). Students who always get the correct answers do not necessarily understand how or why their answers are accurate. Students who fail can develop useful strategies for solving problems and determining where they went wrong (Amidon et al., 2020; Livy et al., 2018; Russo et al., 2021).

Hearing or seeing students' failures will help teachers. When Productive Struggle is happening, it is important the process is documented. The teacher's responses, the ways in which a student attempts to solve a problem, and records of how and why a student becomes "stuck" become important components that will be necessary to consider the next time the lesson is taught (Zeybek, 2016). The goal is to provide students with the necessary tools to independently solve the problems.

It is common for teachers to want to reward their students with free time to do as they wish, but in mathematics, that time can be used to benefit the more efficient students. Ewing et al. (2019)

discuss their study about a teacher who allowed the students to read when they were finished with their math instead of offering more challenging problems. Those students lost an opportunity to build upon their knowledge and gain a deeper mathematical understanding. In contrast, Betts and Rosenberg (2016), describe how students who finished quickly were challenged to find multiple solutions to the same problem. A problem with multiple routes to the solution not only engages higher-achieving students, but also encourages students to use other avenues that support their mathematical strengths.

Time is the key to Productive Struggle. In several studies, it was determined that given adequate time, ability to work in small groups, and an environment where students feel safe, students were able to solve the difficult problems (Ewing et al., 2019; Warshauer, 2015; Zeybek, 2016). Time is imperative for students to develop their own strategies and figure out whether the strategies will work or not. It also provides teachers with an opportunity to identify struggling students and guide them towards the correct path without hindering students' critical thinking.

### **Potential Productive Struggle Problems**

Productive Struggle instruction is not without challenges. If the wrong tasks are chosen, students are likely to grow frustrated and will be unwilling to try again. A teacher that provides too many steps, reduces the cognitive load, or does not provide enough time will undermine Productive Struggle. A teacher who does not allow students to learn from their mistakes is preventing them from building a deeper understanding of mathematics (Ewing et.al., 2019).

Productive Struggle can lead to uncomfortable feelings in students. When a student is tasked with facing the unknown, it will often lead to anxiety (Murdoch et al., 2020). The first few times a student struggles productively, it will be difficult. Prior to this new problem-solving instruction, students were previously taught the specific steps to follow. In the former model, they did not understand what they did, only that they had to follow the identified steps. In Productive Struggle, many students push back, asking for help or saying they do not know what to do. They are unwilling to try because of the anxiety of getting the problem wrong.

Before students learn to productively struggle, many of them perceive their struggle as a weakness. When students observe themselves falling behind their classmates, it is seen as shameful. Shame will cause students to withdraw, lash out at others or themselves, or cause them to avoid the issue (Amidon et al., 2020). If this shame remains unchecked, the students could forever see themselves as "bad" at math. One effective strategy teachers use to deal with shameful feelings is to place less emphasis on correct answers and more focus on the learning process (Amidon et.al., 2020).

The benefits of Productive Struggle outweigh the challenges that may occur. Students develop a deeper understanding of mathematics, including the disparities between what they do and do not know. Even though Productive Struggle may lead to feelings of shame and anxiety in students practicing this method, the benefits include pride and joy in their accomplishments after succeeding in solving the problem. Regarding the time and effort, Productive Struggle evidences value for both teachers and students.

## Parents and Productive Struggle

The effectiveness of Productive Struggle instruction is influenced by parents as well. Many parents were taught mathematics differently than current instruction. This causes parents to grow frustrated with new instruction methods and, sometimes, insist on the ways they are familiar with using. This will only cause problems for the student in the classroom. Once parents understand the benefits of Productive Struggle instruction and why teachers are choosing to implement this method, they will appreciate the advantages for their children.

Unlike teachers, parents are less likely to understand the necessity for struggle. They may not see the struggle as beneficial, only as a waste of time. Parents want their children to succeed, so the parents are likely to talk their child through the process, or just provide the correct answer and move on (Russo et al., 2021). Mothers who helped their children with their homework reported a negative attitude toward the experience. This was caused not by having to help, but because they thought of their child as helpless (Vazquez et al., 2020). This idea is not beneficial to the student or the parent.

Parents react differently to Productive Struggle in their children's homework. Parents who view their children as "good" at mathematics are more likely to allow their students to struggle because they believe their child has the background knowledge to solve difficult problems (Vazquez et al., 2020). Some parents were not content leaving their students alone. Parents with strong stereotypes linked to math are more likely to interfere with their child's homework (Vazquez et al., 2020). In order for Productive Struggle instruction to be effective, a united front in implementation between the parents and the teachers becomes necessary.

It is helpful to inform parents about Productive Struggle instruction and how it includes teachers making time for students to solve the problems themselves. It is important for parents to understand that if a teacher tells the students how to solve the problem, the students lose the opportunity to develop a deeper understanding of the process. Additionally, parents may require assurance that teachers understand how a student's struggle will more effectively prepare them for the next lesson. However, the more a teacher knows about the topic and the moments in the problem-solving process with the highest potential to derail the student, the more effective the instruction will be along with increased opportunity for greater positive outcomes. Instruction in Productive Struggle cannot succeed in the classroom without parental support on the home front. Productive Struggle in the classroom requires many elements working together, including teacher and parent cooperation and understanding, to enable the students to be successful.

## Conclusion

Productive Struggle is an essential method of instruction for mathematics (Amidon et al., 2020; Baker et al., 2020; Ewing et al., 2019; Granberg, 2016; Lemley et al., 2019; Zeybek, 2016). The key to this method of instruction is to allow students to struggle and come to appreciate the benefits of the process. If they do not understand the reasoning behind their struggle, they are more likely to give up on the difficult problems. Students engaged in Productive Struggle realize an additional advantage. Productively, struggling students outperform students with similar



capabilities who did not participate in this problem-solving instruction of Productive Struggle (O'Dell, 2018; Vazquez et al., 2020; Warshauer et al., 2021).

Productive Struggle encourages students to take risks (Livy et al., 2018; Murdoch et al., 2020; Russo et al., 2021). In order to build their own knowledge and understanding, students try out ways to find solutions. These methods may be successful or not, but by engaging in this process, students become more effective at problem solving in the future.

## References

- Amidon, J., Monroe, A., Rock, D. & Cook, C. (2020). Shame, shame, go away: Fostering productive struggle with mathematics. *Kappa Delta Pi Record*, 56(2), 64-69. doi: <https://doi.org/10.1080/00228958.2020.1729636>
- Baker, K., Jessup, N.A., Jacobs, V.R., Empson, S.B., & Case, J. (2020). Productive struggle in action. *Mathematics Teacher*, 113(5), 361-367. doi: [10.5951/mtlt.2019.0060](https://doi.org/10.5951/mtlt.2019.0060)
- Betts, P., & Rosenberg, S. (2016). Making sense of problem solving and productive struggle. *Delta-K*, 53(2), 26–31. <https://search-ebscohost-com.ezproxy.mtsu.edu/login.aspx?direct=true&db=eue&AN=118517581&site=eds-live&scope=site>
- Brousseau, G., & Gibel P. (2005). Didactical handling of students’ reasoning processes in problem solving situations. *Beyond the Apparent Banality of the Mathematics Classroom*. Springer, Boston, MA. doi: [10.1007/0-387-30451-7\\_2](https://doi.org/10.1007/0-387-30451-7_2)
- Ewing, J., Gresham, G. J., & Dickey, B. (2019). Pre-service teachers learning to engage all students, including English language learners, in productive struggle. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 2. <https://eric.ed.gov/?id=EJ1206251>
- Granberg, C. (2016). Discovering and addressing errors during mathematics problem-solving-A productive struggle? *Journal of Mathematical Behavior*, 42, 33–48. doi: 10.1016/j.jmathb.2016.02.002
- Kamii, C. (1982). *Number in preschool and kindergarten*. National Association for the Education of Young Children.
- Kamii, C. (2000). *Young children reinvent arithmetic: Implications of Piaget’s theory* (2<sup>nd</sup> ed.). Teachers College Press.
- Lemley, S. M., Ivy, J. T., Franz, D. P., & Oppenheimer, S. F. (2019). Metacognition and middle grade mathematics teachers: Supporting productive struggle. *Clearing House*, 92(1/2), 15–22. doi: 10.1080/00098655.2018.1547264
- Livy, S., Muir, T., & Sullivan, P. (2018). Challenging tasks lead to productive struggle! *Australian Primary Mathematics Classroom*, 23(1), 19–24. <https://core.ac.uk/download/pdf/211710208.pdf>
- Murdoch, D., English, A. R., Hintz, A., & Tyson, K. (2020). “Feeling heard”: Inclusive education, transformative learning, and productive struggle. *Educational Theory*, 70(5) 653-679. doi: 10.1111/edth.12449
- O’Dell, J. R. (2018). The interplay of frustration and joy: Elementary students’ productive struggle when engaged in unsolved problems. *North American Chapter of the International Group for the Psychology of Mathematics Education*. <https://eric.ed.gov/?id=ED606604>
- Russo, J., Bobis, J., Downton, A., Livy, S., & Sullivan, P. (2021). Primary teacher attitudes towards productive struggle in mathematics in remote learning versus classroom-based settings. *Education Sciences*, 11(35), 1-13. doi: 10.3390/educsci11020035
- Vazquez, S. R., Ermeling, B. A., & Ramirez G. (2020). Parental beliefs on the efficacy of productive struggle and their relation to homework-helping behavior. *Journal for Research in Mathematics Education*, 51(2), 179-203. doi: 10.5951/jresmetheduc-2020-0019

- Warshauer, H. K. (2015). Strategies to support productive struggle. *Mathematics Teaching in the Middle School*, 20(7), 390–393. doi: 10.5951/mathteachmidscho.20.7.0390
- Warshauer, H. K., Starkey, C., & Herrera, C. A. (2021). Developing prospective teachers' noticing and notions of productive struggle with video analysis in a mathematics content course. *Journal of Mathematics Teacher Education*, 24(1), 89-121. doi: 10.1007/s10857-019-09451-2
- Zeybek, Z. (2016). Productive struggle in a geometry class. *International Journal of Research in Education and Science*, 2(2), 396-415. <https://eric.ed.gov/?id=EJ1110272>